DOCUMENT RESUME

ED 349 168 SE 052 981

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TITLE Relationship between Mental Models Related to the Particulate Nature of Matter and the Infinite Nature of Geometrical Figures.
PUB DATE Mar 92
NOTE 10p.; Paper prepared for the Annual Meeting of the National Association for Research in Science Teaching (March 1992).
PUB TYPE Speeches/Conference Papers (150)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS *Cognitive Processes; *Cognitive Structures; Foreign Countries; *Geometric Concepts; Geometry; Mathematics Education; *Matter; *Problem Solving; Schemata (Cognition); Science Education; Secondary Education; Thinking Skills
IDENTIFIERS *National Assn for Research in Science Teaching; *Problem Structure

ABSTRACT

A study was conducted in Israel to determine effects of external similarity in problem structure on students' responses. Fifty students from each of the 7th, 8th, 10th, and 12th grade levels were presented with three problems involving successive divisions that were similar in structure. The problems asked separately whether the processes of continually halving a line segment, a copper wire, or a cup full of water would come to an end. The order of the questioning was varied to counterbalance the effect that order might have. Results indicated that the frequency of responses indicating that the process was infinite increased with grade level. Findings indicate that procedural similarity affected students' responses with 51% of the students giving the same response to all three problems. A study of response patterns revealed that figural similarity minimally affected students' responses with 16% of the students giving finite responses to the line segment and copper wire and an infinite response to the water problem, or vice versa. (MDH)

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# The Relationship between Mental Models Related to the Particulate Nature of Matter and the Infinite Nature of Geometrical Figures

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Paper prepared for NARST, March 1992.

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# The Relationship between Mental Models Related to the Particulate Nature of Matter and the Infinite Nature of Geometrical Figures

## Introduction

Research in science and mathematics education has indicated that students tend to use inappropriate models to solve problems in these subjects (e.g., diSessa, 1983; Fischbein, 1987; McCloskey, 1983). One central issue in science education research concerns the factors which affect students' choice of a model for solving a given problem. More specifically: how do factors related to the problem, i.e., its structure, the numerical data, the figural aspects and its content domain, affect the student's solution, and what effects do factors related to the solver, i.e., age, grade level and instruction, have on his/her solution to a given problem?

Previous studies in science and mathematics education have shown that students tend to use the same model to solve externally similar problems relating to the particulate nature of matter and to potential infinity (Fischbein, Tirosh, Stavy and Oster, 1990; Tirosh and Stavy, in press). Thus, when students were asked to consider two problems, one of which concerned the successive division of a line segment while the other dealt with the successive division of a copper wire, a substantial number of tenth and twelfth grade students majoring in mathematics stated that both processes were endless. The fact that many inexperienced solvers offered this response to the scientific problem can be attributed to their tendency to rely on the external features of problems (Carey, 1985; Chi, Feltovich, and Glazer, 1981; Chi, Glaser, and Rees, 1982; Larkin, 1983; Larkin and Rainhard, 1984; Hardiman, Dufresne and Mestre, 1989).

The two problems presented to the students in the previous study were similar in two aspects: they related to the same process (successive division) and figurally resembled each other (line segment and copper wire). The present study aimed at examining the effect of each of these two aspects of the external similarity on students' responses.

## Method

### Subjects

Two-hundred upper middle-class students participated in this study. Fifty students each from the seventh, eighth, tenth and twelfth grade level were investigated. The tenth and twelfth grade students were studying mathematics as their major subject.

Seventh grade students had experience with elementary school geometry but did not yet receive any instruction in Euclidean geometry. At the time of the research, they had finished studying a chapter on the particulate nature of matter. Eighth grade students had been formally instructed concerning elements, compounds and the periodic table, while in mathematics, they had not yet been

instructed in either geometry or infinite processes. Tenth grade students had studied basic Euclidean geometry (i.e., undefined and defined terms, axioms, postulates, definitions, theorems and proofs); they had not received any additional instruction concerning the structure of matter. Twelfth grade students took an introductory course in calculus in which they dealt with infinite series and limits. They also studied chemistry on a minor level (stoichiometry, the structure of the atom, etc.).

The Problems

The following three problems were presented:

1. The line segment problem

- "Consider a line segment AB. Divide it into two equal parts. Divide one half into two equal parts. Continue dividing it in the same way. Will this process come to an end? Explain your answer."

2. The copper wire problem

- "Consider a piece of copper wire. Divide it into two equal parts. Divide one half into two equal parts. Continue dividing it in the same way. Will this process come to an end? Explain your answer."

3. The water problem

- "Consider a cup full of water. Pour out half of it. Again, pour out half of the water left in the cup. Continue pouring out half of the remaining water in the same way. Will this process come to an end? Explain your answer."

All three problems are externally similar as each of them involve successive division. Two of them (line segment and copper wire) are also figurally similar. However, the first problem (line segment) concerns an ideal, geometrical segment, whereas the other two deal with material substances (copper wire and water). The correct responses are that in the case of the line segment the halving process is endless, whereas in the case of the copper wire and water the halving process stops when the atomic (molecular) level is reached.

Procedure

The three problems were administered to all students during one class period (about 45 minutes). In order to counterbalance the effect of the order of presentation of the problems, half of the students in each grade level received the mathematics problem first while the other half received the science problems first. The mathematics problem was given on one sheet of paper along with other irrelevant mathematics problems. The science problems were given on another sheet of paper along with other irrelevant science problems. Each sheet was collected after the student had responded.

## Results

Responses to the three problems

Two types of responses were given to each of the problems (see Table 1): the process is endless, or the process will come to an end. In each of these problems, the frequency of the infinite responses significantly increased with grade level while that of the finite responses significantly decreased.

Table 1

Responses to the line segment, copper wire and water problems

| | Line segment Grade | | | | Copper wire Grade | | | | Water Grade | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 7 | 8 | 10 | 12 | 7 | 8 | 10 | 12 | 7 | 8 | 10 | 12 |
| *The process is endless - Total* | *30** | *50** | *76** | *72** | *26* | *30* | *48* | *46* | *22* | *12* | *38* | *36* |
| One can always divide by two | 28 | 40 | 46 | 52 | 22 | 26 | 40 | 38 | 22 | 12 | 38 | 36 |
| There is an infinite number of points/atoms | - | 4 | 26 | 20 | - | - | - | - | - | - | - | - |
| Points/atoms can be divided | 2 | 6 | 4 | - | 4 | 4 | 8 | 8 | - | - | - | - |
| *The process will end- Total* | *70* | *50* | *24* | *28* | *74** | *70** | *52** | *54** | *78** | *88** | *62** | *64** |
| It will become extremely small | 42 | 18 | 8 | 10 | 40 | 26 | 24 | 18 | 74 | 74 | 38 | 26 |
| There is a finite number of points/particle | 8 | 22 | 2 | 6 | 8 | 6 | 2 | 8 | - | 4 | 4 | 10 |
| We shall reach a point/particle | 20 | 10 | 14 | 12 | 26 | 38 | 26 | 28 | 4 | 10 | 20 | 28 |

* correct responses

Three types of justifications were given for the "endless" response. The most dominant in all grade levels was: "One can always divide by two." The second most common justification was: "There is an infinite number of points/atoms." The third, less frequent, justification was: "We shall reach a point/atom but it can also be divided."

Students used three types of justifications for their response that the successive division processes would come to an end. A substantial number of seventh and eighth grade students argued that "we will not be able to divide anymore because it will become extremely small." The second most common justification was that "there is a finite number of points/particles." The third type of justification was: "we shall not be able to divide anymore as we shall reach a point/particle."

Patterns of responses to the three problems

The eight possible response patterns are presented in Table 2. Students who gave the same response to all problems (either finite or infinite) are included in the concordant patterns. Those who gave different answers to the problems are included in the discordant patterns.

Table 2

Response patterns to the line segment, copper wire and water problems

| | | | Grade | | | |
|---|---|---|---|---|---|---|
| Segment | Wire | Water | 7 | 8 | 10 | 12 |
| I *Concordant patterns - Total* | | | *74* | *40* | *44* | *48* |
| a. finite | finite | finite | 60 | 38 | 18 | 18 |
| b. infinite | infinite | infinite | 14 | 2 | 26 | 30 |
| II *Discordant patterns - Total* | | | *26* | *60* | *56* | *52* |
| a. Correct pattern | | | | | | |
| infinite | finite | finite | 8 | 24 | 24 | 32 |
| b. Reversed pattern | | | | | | |
| finite | infinite | infinite | 2 | 2 | 2 | 2 |
| c. Figural patterns | | | | | | |
| finite | finite | infinite | 4 | 4 | 2 | 2 |
| infinite | infinite | finite | 6 | 20 | 18 | 8 |
| d. Non-figural patterns | | | | | | |
| finite | infinite | finite | 4 | 6 | 2 | 6 |
| infinite | finite | infinite | 2 | 4 | 8 | 2 |

Concordant Patterns. The concordant patterns reflect the same response to all problems. The only aspect common to all three problems is the process of successive division. Thus, the percentage of students included under the two concordant patterns gives an indication of the effect of the procedural similarity on students' responses to the problems. Table 2 shows that a substantial number of students at all grade levels responded similarly to all three problems: either that the process of successive division would come to an end (the finite-finite-finite pattern- Ia on Table 2) or that the process was endless (the infinite-infinite-infinite pattern- Ib on Table 2). Nearly three-quarters of the seventh graders (74%) gave the same response to all problems. From the eighth grade onwards, the percentage of students giving the same response was between 40% and 48% .

The frequency of the finite-finite-finite pattern significantly decreased from grade 7 to 10 while that of the infinite-infinite-infinite pattern significantly increased over the same grades. The most pronounced increase in the infinite-infinite-infinite pattern appears from grade 8 to grade 10. The decrease in the finite-finite-finite response pattern indicates that students in the upper grade levels gave up their initial, adequate, finite response to the copper wire and to the water problems. The increase in the infinite-infinite-infinite pattern indicates the development of a general view of successive division processes as endless.

It is interesting to note that almost all the students who showed a concordant response pattern also offered the same justification for all problems.

Discordant patterns. The discordant patterns can be divided into four groups.

*The correct pattern (IIa).* This response pattern was the most frequent among the discordant patterns. Although the frequency of this correct response significantly increased with grade level, even in the highest grade no more than 32% of the students evinced it. *The reversed pattern (IIb).* A finite response to the line segment problem and an infinite response to the copper wire and the water problems. This pattern was produced by very few students at each grade level. *Figural patterns (IIc).* Students who gave the same response to the line segment and the copper wire problems but a different response to the water problem are included in the two figural patterns. The percentage of these students gives an indication of the effect of the figural similarity (segment and wire) on students' responses. *Non-figural patterns (IId).* Students who gave the same response to the line segment and the water problems and a different response to the copper wire problem are included in the two nonfigural patterns. The frequency of these patterns at each grade level did not exceed 10% .

## Discussion

Our data indicate that the procedural similarity had a profound effect on students' responses. In fact, 51% of the entire population gave the same response (either finite or infinite) to all problems. Figural similarity also affected students' responses, but to a much lower extent: only 16% of the population exhibited the figural patterns of response (either finite responses to the line segment and copper wire problems and an infinite response to the water problem, or vice versa).

In our previous studies (Fischbein, Tirosh, Stavy, and Oster, 1991; Tirosh and Stavy, in press), which dealt only with the line segment and the copper wire problems, it was suggested that the figural similarity between the two problems was the dominant factor triggering similar responses. However, the present study, which included an additional problem involving water, which is figurally different from the line segment and the copper wire, indicates that the procedural and not the figural similarity was the most dominant factor triggering concordant responses. The predominant effect of the procedural similarity will be further elaborated in the presentation.

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